

Theorem If  $\{a_n\}$  is any sequence then

Examples on  $\inf$  and  $\sup$

Ex 1) If  $a_n = \frac{\sin n\pi}{3}$ ,  $n \in \mathbb{N}$ ,

then the sequence  $\{a_n\}$  is

$$\left\{ \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, 0, -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, 0, \frac{\sqrt{3}}{2} \right\}$$

$\inf = -\frac{\sqrt{3}}{2}$  and  $\sup = \frac{\sqrt{3}}{2}$  for each

$n \in \mathbb{N}$ .  $\inf a_n = -\frac{\sqrt{3}}{2}$  and  $\sup a_n = \frac{\sqrt{3}}{2}$

If  $a_n = \frac{\sin n\pi}{3} + \left(\frac{2}{n}\right)^n$ ,  $n \in \mathbb{N}$ , then show that  $\inf a_n = -1$  and  $\sup a_n = 1$

The sequence  $\{a_n\}$  is bounded for  $\frac{1}{2} \leq$

$a_n \leq 1$  the terms of the sequence is given by  $a_{2n} = \frac{1}{2^{2n}}$  for all  $n \in \mathbb{N}$

$$a_{4n+1} = 1 - \frac{1}{4^{2n+1}} \quad \forall n$$

$$a_{4n+3} = -1 + \frac{1}{4^{2n+3}} \quad \forall n$$

Let  $\inf a_n = l$  and  $\sup a_n = M$