$\frac{}{2n} = \frac{1}{2\infty} = 0$

Q (4) If $a_n = (-1)^n \left(1 + \frac{1}{n}\right)$, $n \in N$.

Then $\underline{A_n} = \inf \left\{ (-1)^n \left(1+\frac{1}{n}\right), (-1)^{n+1}\left(1+\frac{1}{n+1}\right), \ldots \right\}$

$$= \begin{cases} -\left(1+\frac{1}{n}\right) & \text{if } n \text{ is odd} \\ -1\left(1+\frac{1}{n+1}\right) & \text{if } n \text{ is even} \end{cases}$$

$\underline{\lim}\, a_n = \sup \underline{A_n} = \sup \left\{ -2, -\frac{4}{3}, -\frac{6}{5}, \ldots \right\}$

$= -1$

and $\overline{\lim}\, a_n = 1$

Th. (7) If $\{a_n\}$ is any sequence, then

$\underline{\lim}\, a_n = -\infty$ if and only if $\{a_n\}$ is not bounded below

and $\overline{\lim}\, a_n = +\infty$ if and only if $\{a_n\}$ is not bounded above.

Let $\underline{A_n} = \inf \{ a_n, a_{n+1}, \ldots \}$.

and $\overline{A_n} = \sup \{ a_n, a_{n+1}, \ldots \}$. $n \in N$.

By definition we have $\underline{\lim}\, a_n = -\infty \Leftrightarrow \sup \{ \underline{A_1}, \underline{A_2}, \ldots \} = -\infty$

$\Leftrightarrow \underline{A_n} = -\infty \ \forall n \in N \Leftrightarrow \underline{A_n} = -\infty \ \forall n \in N$.

$\Leftrightarrow \inf \{ a_n, a_{n+1}, \ldots \} = -\infty, \ \forall n \in N$.

$\Leftrightarrow \{a_n\}$ is not bounded below.

for limit superior $\overline{A_n} = \sup \{ a_n, a_{n+1}, \ldots \}, n \in N$

By definition $\overline{\lim}\, a_n = +\infty \Leftrightarrow \inf \{ \overline{A_1}, \overline{A_2}, \ldots \} = +\infty$

$\Leftrightarrow \overline{A_n} = +\infty \ \forall n \in N$. $\ldots \sup \{$